1 Derivation of equilibrium conditions

In this appendix, we show how to obtain the first order conditions stated in the main text of the paper from the share equations. Solving

$$
\begin{aligned}
s_1 &= \frac{1}{2} + b + \gamma\left(N_1^a - N_2^a\right) + \psi\left(N_1^c - N_2^c\right) - \beta\left(p_1 - p_2\right) + \varepsilon_1 - \varepsilon_2 && (1)\\
s_2 &= 1 - s_1 && (2)\\
n_1^a &= \frac{1}{2} + \phi + \rho\left(S_1 - S_2\right) - \eta\left(a_1 - a_2\right) + \xi_1 - \xi_2 && (3)\\
n_2^a &= 1 - n_1^a && (4)
\end{aligned}
$$

implies the reduced form share functions

$$
\begin{aligned}
s_1 &= \frac{\frac{1}{2} + \psi\left(N_1^c - N_2^c\right) - \beta\left(p_1 - p_2\right) + b + \varepsilon_1 - \varepsilon_2 + 2\gamma N\left(\phi - \rho S + \xi_1 - \xi_2 - \eta\left(a_1 - a_2\right)\right)}{1 - 4\rho\gamma NS} && (5)\\
n_1^a &= \frac{\frac{1}{2} - \eta\left(a_1 - a_2\right) + \phi + \xi_1 - \xi_2 + 2\rho S\left(b - \gamma N + \varepsilon_1 - \varepsilon_2 - \beta\left(p_1 - p_2\right) + \psi\left(N_1^c - N_2^c\right)\right)}{1 - 4\rho\gamma NS} && (6)
\end{aligned}
$$

with $s_2 = 1 - s_1$ and $n_2^a = 1 - n_1^a$. Provided $4\gamma\rho NS < 1$, these share functions are well behaved. Substituting these share functions into

$$
\Pi_i = \left(p_i - f_i\right) S_i + \left(a_i - c_i\right) N_i^a - d_i\left(N_i^c\right)^2 - F_i
$$

and taking derivatives with respect to p_i, a_i and N_i^c we get the first order conditions:

$$
\begin{aligned}
\left(1 - 4\gamma\rho NS\right) s_1 - \beta\left(p_1 - f_1\right) - 2\rho\beta\left(a_1 - c_1\right) N &= 0 && (7)\\
\left(1 - 4\gamma\rho NS\right) n_1^a - \eta\left(a_1 - c_1\right) - 2\gamma\eta\left(p_1 - f_1\right) S &= 0 && (8)\\
\psi\left(p_1 - f_1 + 2\rho\left(a_1 - c_1\right) N\right) S - 2\left(1 - 4\gamma\rho NS\right) d_1 N_1^c &= 0 && (9)\\
\left(1 - 4\gamma\rho NS\right) s_2 - \beta\left(p_2 - f_2\right) - 2\rho\beta\left(a_2 - c_2\right) N &= 0 && (10)\\
\left(1 - 4\gamma\rho NS\right) n_2^a - \eta\left(a_2 - c_2\right) - 2\gamma\eta\left(p_2 - f_2\right) S &= 0 && (11)\\
\psi\left(p_2 - f_2 + 2\rho\left(a_2 - c_2\right) N\right) S - 2\left(1 - 4\gamma\rho NS\right) d_2 N_2^c &= 0. && (12)
\end{aligned}
$$

Define $\Delta = 1 - 4\gamma\rho NS$. Then (7) implies

$$
s_1\Delta - \beta\left(p_1 - f_1\right) - 2\rho\beta\left(a_1 - c_1\right) N = 0 \tag{13}
$$

and (8) implies

$$
a_1 - c_1 = \frac{n_1^a\Delta}{\eta} - 2\gamma\left(p_1 - f_1\right) S. \tag{14}
$$

Substituting (14) into (13) implies

$$
s_1\Delta - \beta\left(p_1 - f_1\right) - 2\rho\beta\left(\frac{n_1^a\Delta}{\eta} - 2\gamma\left(p_1 - f_1\right) S\right) N = 0
$$

which implies

$$\beta\left(1-4\gamma\rho NS\right)\left(p_1-f_1\right)=s_1\Delta-\frac{2\rho\beta N_1^a\Delta}{\eta}.$$

Replacing $1-4\gamma\rho NS$ by Δ, and solving for p_1-f_1 gives the equilibrium condition

$$p_1-f_1=\frac{s_1}{\beta}-\frac{2\rho N_1^a}{\eta}. \tag{15}$$

The other equilibrium conditions for p_2-f_2, a_1-c_1 and a_2-c_2 are derived in exactly the same fashion. Finally, to get the first order conditions on content, use (9) and substitute in the equilibrium conditions for p_1-f_1 and a_1-c_1 and likewise for (12) substitute in the equilibrium conditions for p_2-f_2 and a_2-c_2.

2 Full comparative static results

If we define

$$\Delta=9\eta-\left(8\beta\rho^2NS+20\gamma\rho\eta NS+\frac{8\gamma^2\eta^2NS}{\beta}+\frac{3\psi^2\eta S}{\beta d}\right)$$

the full comparative static results allowing for network effects are:

$$\begin{aligned}
\frac{dp_1}{db} &= -\frac{dp_2}{db}=\frac{\left(3\eta-4\rho^2\beta NS-8\rho\gamma\eta NS\right)}{\beta\Delta}\\
\frac{ds_1}{db} &= -\frac{ds_2}{db}=\frac{3\eta}{\Delta}\\
\frac{da_1}{db} &= -\frac{da_2}{db}=\frac{2\left(\rho-\frac{\gamma\eta}{\beta}\right)S}{\Delta}\\
\frac{dn_1^a}{db} &= -\frac{dn_2^a}{db}=\frac{2\eta\left(\rho+\frac{2\gamma\eta}{\beta}\right)S}{\Delta}\\
\frac{dN_1^c}{db} &= -\frac{dN_2^c}{db}=\frac{3\psi\eta S}{2\beta d\Delta}
\end{aligned}$$

$$\begin{aligned}
\frac{dp_1}{d\phi} &= -\frac{dp_2}{d\phi}=\frac{2\eta\left(\gamma-\frac{\beta\rho}{\eta}+\frac{\rho\psi^2S}{\eta d}\right)N}{\beta\Delta}\\
\frac{ds_1}{d\phi} &= -\frac{ds_2}{d\phi}=\frac{2\left(2\beta\rho+\gamma\eta\right)N}{\Delta}\\
\frac{da_1}{d\phi} &= -\frac{da_2}{d\phi}=\frac{3\beta-\left(8\gamma\rho\beta NS+4\gamma^2\eta NS+\frac{\psi^2S}{d}\right)}{\beta\Delta}\\
\frac{dn_1^a}{d\phi} &= -\frac{dn_2^a}{d\phi}=\frac{\eta\left(3\beta-\frac{\psi^2S}{d}\right)}{\beta\Delta}\\
\frac{dN_1^c}{d\phi} &= \frac{dN_2^c}{d\phi}=\frac{\psi\left(2\rho\beta+\gamma\eta\right)NS}{\beta d\Delta}
\end{aligned}$$

$$
\begin{aligned}
\frac{dp_1}{df_1} &= \frac{6\beta\eta - \left(4\rho^2\beta^2 NS + 12\rho\gamma\eta\beta NS + 8\gamma^2\eta^2 NS + \frac{3\psi^2\eta S}{d}\right)}{\beta\Delta} \\
\frac{dp_2}{df_1} &= \frac{3\eta - \left(4\rho^2\beta NS + 8\rho\gamma\eta NS\right)}{\Delta} \\
\frac{ds_1}{df_1} &= -\frac{ds_2}{df_1} = -\frac{3\beta\eta}{\Delta} \\
\frac{da_1}{df_1} &= -\frac{da_2}{df_1} = \frac{2\left(\gamma\eta - \beta\rho\right)S}{\Delta} \\
\frac{dn_1^a}{df_1} &= -\frac{dn_2^a}{df_1} = -\frac{2\eta\left(\beta\rho + 2\gamma\eta\right)S}{\Delta} \\
\frac{dN_1^c}{df_1} &= -\frac{dN_2^c}{df_1} = -\frac{3}{2}\frac{\psi\eta S}{d\Delta}
\end{aligned}
$$

$$
\begin{aligned}
\frac{dp_1}{dc_1} &= -\frac{dp_2}{dc_1} = \frac{2\eta\left(\rho - \frac{\eta\gamma}{\beta} - \frac{\rho\psi^2 S}{\beta d}\right)N}{\Delta} \\
\frac{ds_1}{dc_1} &= -\frac{ds_2}{dc_1} = -\frac{2\eta\left(2\beta\rho + \gamma\eta\right)N}{\Delta} \\
\frac{da_1}{dc_1} &= \frac{6\eta\beta - \left(12\gamma\rho\eta\beta NS + 8\beta^2\rho^2 NS + 4\gamma^2\eta^2 NS + \frac{2\psi^2\eta S}{d}\right)}{\beta\Delta} \\
\frac{da_2}{dc_1} &= \frac{3\eta\beta - \left(8\gamma\rho\eta\beta NS + 4\gamma^2\eta^2 NS + \frac{\psi^2\eta S}{d}\right)}{\beta\Delta} \\
\frac{dn_1^a}{dc_1} &= -\frac{dn_2^a}{dc_1} = -\frac{\eta^2\left(3\beta - \frac{\psi^2 S}{d}\right)}{\beta\Delta} \\
\frac{dN_1^c}{dc_1} &= -\frac{dN_2^c}{dc_1} = -\frac{\psi\eta\left(2\rho\beta + \gamma\eta\right)NS}{\beta d\Delta}
\end{aligned}
$$

3 Derivation of elasticities with multihoming

Recall the following definitions in the text:

$$
\begin{aligned}
s_i^L &= \frac{R_i}{R_1 + R_2} \\
n_i^L &= \frac{A_i}{A_1 + A_2}
\end{aligned}
$$

where R_i is exclusive readers and A_i is exclusive advertisers. We work out each elasticity in turn. We note for below that using the definitions of λ_R and S_i, we can rewrite s_1^L in terms of observables (and likewise for n_1^L) so that

$$\begin{aligned} s_i^L &= \frac{(1+\lambda_R)\, s_i - \lambda_R}{1-\lambda_R} \\ n_i^L &= \frac{(1+\lambda_A)\, n_i^a - \lambda_A}{1-\lambda_A}. \end{aligned}$$

3.1 Elasticity of readership share to content share

This is the elasticity

$$\frac{ds_i^L}{dn_i^c}\frac{n_i^c}{s_i^L}$$

Then

$$\begin{aligned} \frac{ds_i^L}{dn_i^c}\frac{n_i^c}{s_i^L} &= \frac{ds_i^L}{ds_i}\frac{ds_i}{dN_i^c}\frac{dN_i^c}{dn_i^c}\frac{n_i^c}{s_i^L} \\ &= \left(\frac{1+\lambda_R}{1-\lambda_R}\right)\psi\left(\frac{1-\lambda_R}{1+\lambda_R}\right)N_i^c\frac{1}{\left(\frac{(1+\lambda_R)s_i-\lambda_R}{1-\lambda_R}\right)} \\ \frac{ds_i^L}{dn_i^c}\frac{n_i^c}{s_i^L} &= \frac{\psi N_i^c}{\left(\frac{(1+\lambda_R)s_i-\lambda_R}{1-\lambda_R}\right)} \end{aligned}$$

where we have used the adjusted share equation

$$s_1 = \frac{1}{2} + \left(\frac{1-\lambda_R}{1+\lambda_R}\right)\left(b + \gamma\left(N_1^a - N_2^a\right) + \psi\left(N_1^c - N_2^c\right) - \beta\left(p_1 - p_2\right) + \varepsilon_1 - \varepsilon_2\right).$$

3.2 Elasticity of readership share to advertising share

This is the elasticity

$$\frac{ds_i^L}{dn_i^L}\frac{n_i^L}{s_i^L}.$$

Then

$$\begin{aligned} \frac{ds_i^L}{dn_i^L}\frac{n_i^L}{s_i^L} &= \frac{ds_i^L}{ds_i}\frac{ds_i}{dN_i^a}\frac{dN_i^a}{dn_i^a}\frac{dn_i^a}{dn_i^L}\frac{n_i^L}{s_i^L} \\ &= \left(\frac{1+\lambda_R}{1-\lambda_R}\right)\gamma\left(\frac{1-\lambda_R}{1+\lambda_R}\right)N\left(\frac{1-\lambda_A}{1+\lambda_A}\right)\frac{\left(\frac{(1+\lambda_A)n_i^a-\lambda_A}{1-\lambda_A}\right)}{\left(\frac{(1+\lambda_R)s_i-\lambda_R}{1-\lambda_R}\right)} \\ \frac{ds_i^L}{dn_i^L}\frac{n_i^L}{s_i^L} &= \gamma N\frac{\left(\frac{(1+\lambda_A)n_i^a-\lambda_A}{1+\lambda_A}\right)}{\left(\frac{(1+\lambda_R)s_i-\lambda_R}{1-\lambda_R}\right)} \end{aligned}$$

where we have used the adjusted share equation

$$s_1 = \frac{1}{2} + \left(\frac{1-\lambda_R}{1+\lambda_R}\right)\left(b + \gamma\left(N_1^a - N_2^a\right) + \psi\left(N_1^c - N_2^c\right) - \beta\left(p_1 - p_2\right) + \varepsilon_1 - \varepsilon_2\right).$$

3.3 Elasticity of readership share to cover price

This is the elasticity

$$\frac{ds_i^L}{dp_i}\frac{p_i}{s_i^L}.$$

Then

$$\begin{aligned}\frac{ds_i^L}{dp_i}\frac{p_i}{s_i^L} &= \frac{ds_i^L}{ds_i}\frac{ds_i}{dp_i}\frac{p_i}{s_i^L} \\ &= -\left(\frac{1+\lambda_R}{1-\lambda_R}\right)\beta\left(\frac{1-\lambda_R}{1+\lambda_R}\right)\frac{p_i}{\left(\frac{(1+\lambda_R)s_i-\lambda_R}{1-\lambda_R}\right)} \\ \frac{ds_i^L}{dp_i}\frac{p_i}{s_i^L} &= -\frac{\beta p_i}{\left(\frac{(1+\lambda_R)s_i-\lambda_R}{1-\lambda_R}\right)}\end{aligned}$$

where we have used the adjusted share equation

$$s_1 = \frac{1}{2}+\left(\frac{1-\lambda_R}{1+\lambda_R}\right)\left(b+\gamma\left(N_1^a-N_2^a\right)+\psi\left(N_1^c-N_2^c\right)-\beta\left(p_1-p_2\right)+\varepsilon_1-\varepsilon_2\right).$$

3.4 Elasticity of advertising share to readership share

This is the elasticity

$$\frac{dn_i^L}{ds_i^L}\frac{s_i^L}{n_i^L}.$$

Then

$$\begin{aligned}\frac{dn_i^L}{ds_i^L}\frac{s_i^L}{n_i^L} &= \frac{dn_i^L}{dn_i^a}\frac{dn_i}{dS_i}\frac{dS_i}{ds_i}\frac{ds_i}{ds_i^L}\frac{s_i^L}{n_i^L} \\ &= \left(\frac{1+\lambda_A}{1-\lambda_A}\right)\rho\left(\frac{1-\lambda_A}{1+\lambda_A}\right)S\left(\frac{1-\lambda_R}{1+\lambda_R}\right)\frac{\left(\frac{(1+\lambda_R)s_i-\lambda_R}{1-\lambda_R}\right)}{\left(\frac{(1+\lambda_A)n_i^a-\lambda_A}{1-\lambda_A}\right)} \\ \frac{dn_i^L}{ds_i^L}\frac{s_i^L}{n_i^L} &= \rho S\frac{\left(\frac{(1+\lambda_R)s_i-\lambda_R}{1+\lambda_R}\right)}{\left(\frac{(1+\lambda_A)n_i^a-\lambda_A}{1-\lambda_A}\right)}\end{aligned}$$

where we have used the adjusted share equation

$$n_1^a = \frac{1}{2}+\left(\frac{1-\lambda_A}{1+\lambda_A}\right)\left(\phi+\rho\left(S_1-S_2\right)-\eta\left(a_1-a_2\right)+\xi_1-\xi_2\right).$$

3.5 Elasticity of advertising share to ad price

This is the elasticity

$$\frac{dn_i^L}{da_i}\frac{a_i}{n_i^L}$$

Then

$$
\begin{aligned}
\frac{dn_i^L}{da_i}\frac{a_i}{n_i^L} &= \frac{dn_i^L}{dn_i^a}\frac{dn_i}{da_i}\frac{a_i}{n_i^L} \\
&= -\left(\frac{1+\lambda_A}{1-\lambda_A}\right)\eta\left(\frac{1-\lambda_A}{1+\lambda_A}\right)\frac{a_i}{\left(\frac{(1+\lambda_A)n_i^a-\lambda_A}{1-\lambda_A}\right)} \\
\frac{dn_i^L}{da_i}\frac{a_i}{n_i^L} &= -\frac{\eta a_i}{\left(\frac{(1+\lambda_A)n_i^a-\lambda_A}{1-\lambda_A}\right)}
\end{aligned}
$$

where we have used the adjusted share equation

$$
n_1^a = \frac{1}{2} + \left(\frac{1-\lambda_A}{1+\lambda_A}\right)\left(\phi + \rho\left(S_1 - S_2\right) - \eta\left(a_1 - a_2\right) + \xi_1 - \xi_2\right).
$$