

## Comparative static results.

If we define

$$\Delta = 9\eta - \left( 8\beta\rho^2NS + 20\gamma\rho\eta NS + \frac{8\gamma^2\eta^2NS}{\beta} + \frac{3\psi^2\eta S}{\beta d} \right)$$

the full comparative static results allowing for network effects are:

$$\begin{aligned} \frac{dp_1}{db} &= -\frac{dp_2}{db} = \frac{(3\eta - 4\rho^2\beta NS - 8\gamma\rho\eta NS)}{\beta\Delta} \\ \frac{ds_1}{db} &= -\frac{ds_2}{db} = \frac{3\eta}{\Delta} \\ \frac{da_1}{db} &= -\frac{da_2}{db} = \frac{2\left(\rho - \frac{\gamma\eta}{\beta}\right)S}{\Delta} \\ \frac{dn_1^a}{db} &= -\frac{dn_2^a}{db} = \frac{2\eta\left(\rho + \frac{2\gamma\eta}{\beta}\right)S}{\Delta} \\ \frac{dN_1^c}{db} &= -\frac{dN_2^c}{db} = \frac{3\psi\eta S}{2\beta d\Delta} \end{aligned}$$

$$\begin{aligned} \frac{dp_1}{d\phi} &= -\frac{dp_2}{d\phi} = \frac{2\eta\left(\gamma - \frac{\beta\rho}{\eta} + \frac{\rho\psi^2S}{\eta d}\right)N}{\beta\Delta} \\ \frac{ds_1}{d\phi} &= -\frac{ds_2}{d\phi} = \frac{2(2\beta\rho + \gamma\eta)N}{\Delta} \\ \frac{da_1}{d\phi} &= -\frac{da_2}{d\phi} = \frac{3\beta - \left(8\gamma\rho\beta NS + 4\gamma^2\eta NS + \frac{\psi^2S}{d}\right)}{\beta\Delta} \\ \frac{dn_1^a}{d\phi} &= -\frac{dn_2^a}{d\phi} = \frac{\eta\left(3\beta - \frac{\psi^2S}{d}\right)}{\beta\Delta} \\ \frac{dN_1^c}{d\phi} &= \frac{dN_2^c}{d\phi} = \frac{\psi(2\beta\rho + \gamma\eta)NS}{\beta d\Delta} \end{aligned}$$

$$\begin{aligned} \frac{dp_1}{df_1} &= \frac{6\beta\eta - \left(4\rho^2\beta^2NS + 12\gamma\rho\eta\beta NS + 8\gamma^2\eta^2NS + \frac{3\psi^2\eta S}{d}\right)}{\beta\Delta} \\ \frac{dp_2}{df_1} &= \frac{3\eta - (4\rho^2\beta NS + 8\gamma\rho\eta NS)}{\Delta} \\ \frac{ds_1}{df_1} &= -\frac{ds_2}{df_1} = -\frac{3\beta\eta}{\Delta} \\ \frac{da_1}{df_1} &= -\frac{da_2}{df_1} = \frac{2(\gamma\eta - \beta\rho)S}{\Delta} \\ \frac{dn_1^a}{df_1} &= -\frac{dn_2^a}{df_1} = -\frac{2\eta(\beta\rho + 2\gamma\eta)S}{\Delta} \end{aligned}$$

$$\frac{dN_1^c}{df_1} = -\frac{dN_2^c}{df_1} = -\frac{3}{2} \frac{\psi\eta S}{d\Delta}$$

$$\frac{dp_1}{dc_1} = -\frac{dp_2}{dc_1} = \frac{2\eta \left( \rho - \frac{\eta\gamma}{\beta} - \frac{\rho\psi^2 S}{\beta d} \right) N}{\Delta}$$

$$\frac{ds_1}{dc_1} = -\frac{ds_2}{dc_1} = -\frac{2\eta (2\beta\rho + \gamma\eta) N}{\Delta}$$

$$\frac{da_1}{dc_1} = \frac{6\eta\beta - \left( 12\gamma\rho\eta\beta NS + 8\beta^2\rho^2 NS + 4\gamma^2\eta^2 NS + \frac{2\psi^2\eta S}{d} \right)}{\beta\Delta}$$

$$\frac{da_2}{dc_1} = \frac{3\eta\beta - \left( 8\gamma\rho\eta\beta NS + 4\gamma^2\eta^2 NS + \frac{\psi^2\eta S}{d} \right)}{\beta\Delta}$$

$$\frac{dn_1^a}{dc_1} = -\frac{dn_2^a}{dc_1} = -\frac{\eta^2 \left( 3\beta - \frac{\psi^2 S}{d} \right)}{\beta\Delta}$$

$$\frac{dN_1^c}{dc_1} = -\frac{dN_2^c}{dc_1} = -\frac{\psi\eta (2\rho\beta + \gamma\eta) NS}{\beta d\Delta}$$